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NATIONAL WEATHER SERVICE
NATIONAL METEOROLOGICAL CENTER

OFFICE NOTE 246

Objective Determination of the Tropopause Using WMO Operational Definitions

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This is an unreviewed manuscript, primarily intended for informal exchange of information among NMC staff members.

INTRODUCTION:

This Office Note describes a numerical method of objectively determining approximate tropopause pressure and temperature values using only mandatory height and temperature data. The numerical method to be described here is a revised version of the method that is briefly outlined in an article that was presented by W.L. Hughes (Ref. 1) before the International Civil Aviation Organization (ICAO) Area Forecast Panel in Montreal, Canada.

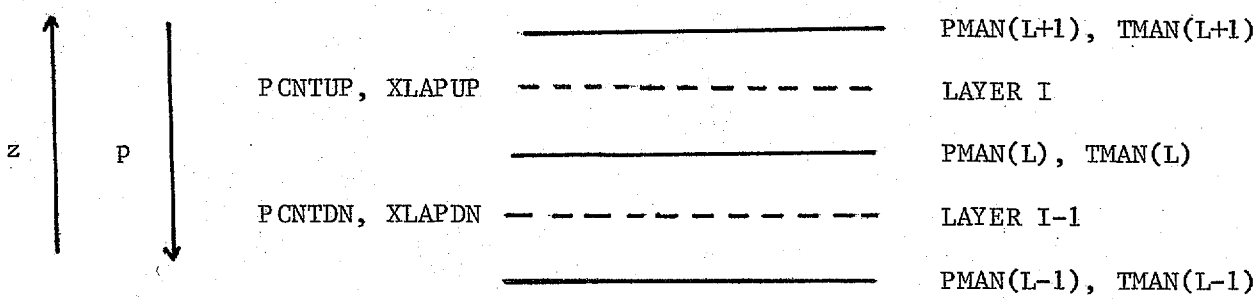
The major premise upon which this proposed numerical method rests is the WMO operational definition for the tropopause. That definition states that the conventional tropopause is the lowest level at which the lapse-rate decreases to 0.002 $^{\circ}C/m$ or less, and the average lapse-rate from this level to any level within the next higher 2000 m does not exceed 0.002 $^{\circ}C/m$ (Refs. 2-3).

MODEL DESCRIPTION:

Using only the mandatory height and temperature data (i.e. 500 - 50 mb), simple lapse-rates (i.e. $\gamma = \Delta T/\Delta z$) are computed for each layer. The lapse-rates are assumed to vary linearly with pressure and, for modelling purposes, are taken to be valid at the central pressure of each layer (Ref. 1). Mandatory data below 500 mb are not used in order to avoid the possibility of misinterpreting a low-level inversion for the level of the tropopause.

After each layer lapse-rate for a radiosonde or gridpoint sounding has been computed, the sounding is scanned in the direction of decreasing pressure until a layer is encountered wherein the lapse-rate is less than 0.002 $^{\circ}C/m$. Whenever this occurs, the central pressure of that layer (i.e. Layer I) is assumed to be above the tropopause and the central pressure of the previous or lower layer (i.e. Layer I-1) is assumed to be below the tropopause[1] (See Figure 1). If, however, the tropopause cannot be bounded in this fashion, or if the computed tropopause pressure is less than 85 mb, then the tropopause pressure is set equal to 85 mb and the temperature at that level is determined by interpolation. In all of the tests that were conducted on ADP file radiosonde data, the critical stability criterion of 0.002 $^{\circ}$C/m was satisfied and the tropopause pressure successfully bracketed.

Once Layer I and Layer I-1 have been determined, the tropopause pressure (PTROPE) may be computed by linear interpolation between the central pressures of these two layers to the level at which the lapse-rate is equal to 0.002 $^{\circ}$C/m. The diagram and list of defined variables below should be referenced to clarify the method and the terms of the interpolation.



DEFINITION OF VARIABLE NAMES:

PMAN(L-1) - The pressure of the higher pressure mandatory level of Layer I-1.

PMAN(L) - Simultaneously, the pressure of the lower pressure mandatory level of Layer I-1 and the pressure of the higher pressure mandatory level of Layer I.

PMAN(L+1) - The pressure of the lower pressure mandatory level of Layer I.

TMAN(L-1) - The mandatory temperature at the higher pressure mandatory level of Layer I-1.

1. Because the lapse-rates are assumed to be valid at the central pressure of each layer, a tropopause found to be below the midpoint of the lowest layer tested would have its pressure set equal to 450 mb and the tropopause temperature at 450 mb would be determined by interpolation between the 500 and the 400 mb levels.

TMAN(L) - Simultaneously, the mandatory temperature at the lower pressure mandatory level of Layer I-1 and at the higher pressure mandatory level of Layer I.

TMAN(L+1) - The mandatory temperature at the lower pressure mandatory level of Layer I.

PCNTDN - The central pressure of Layer I-1.

PCNTUP - The central pressure of Layer I.

XLAPDN - The lapse-rate valid at the central pressure of Layer I-1.

XLAPUP - The lapse-rate valid at the central pressure of Layer I.

If it is assumed, as in the Hughes article, that the lapse-rate varies linearly with pressure, then PTROPE may be determined as follows,

$$\mathrm{PTROPE} = \mathrm{PCNTUP} + \{(0.002 - \mathrm{XLAPUP})(\mathrm{PCNTDN} - \mathrm{PCNTUP})\}/(\mathrm{XLAPDN} - \mathrm{XLAPUP}) \qquad (1)$$

If PTROPE is equal to PMAN(L), then the tropopause temperature (TTROPE) is equal to TMAN(L). If, however, PTROPE is not equal to PMAN(L), then the tropopause temperature must be determined by the method of linear interpolation as expressed in the following equations wherein temperature is a function of the logarithm of pressure;

$$\mathrm{TTROPE} = \mathrm{TMAN(L+1)} + \{(\ln \mathrm{PTROPE} - \ln \mathrm{PMAN(L+1)})(\mathrm{TMAN(L)} - \mathrm{TMAN(L+1)})\}/(\ln \mathrm{PMAN(L)} - \ln \mathrm{PMAN(L+1)}), \text{ for } \mathrm{PTROPE} < \mathrm{PMAN(L)} \qquad (2a)$$

and

$$\mathrm{TTROPE} = \mathrm{TMAN(L)} + \{(\ln \mathrm{PTROPE} - \ln \mathrm{PMAN(L)})(\mathrm{TMAN(L+1)} - \mathrm{TMAN(1)})\}/(\ln \mathrm{PMAN(L-1)} - \ln \mathrm{PMAN(L)}), \text{ for } \mathrm{PTROPE} > \mathrm{PMAN(L)} \qquad (2b)$$

MODEL TESTS:

The model described above as well as two variations of that model were compared with the NMC operational model (See NMC Technical Memos #30 and #33). Operational radiosonde observations served as the source of model input data. In the case of each radiosonde report that was scanned, the tropopause pressure computed by each one of the four models was compared with the pressure of the lowest level tropopause that was reported for each radiosonde. This method of comparing the tropopause values was consistent with the WMO definition of the conventional tropopause mentioned in the introduction. The new model required complete mandatory level data from 500 - 50 mb for both heights and temperatures, while the operational model required the complete mandatory level temperature data only, but from 850 - 50 mb.

One of the variations of the new model consisted of adding the 2000 m extension test cited in the introduction, while a second variation involved changing from a linear dependence between the lapse-rate and pressure to a linear dependence between the lapse-rate and the logarithm of pressure. The second of the two changes was prompted by the fact that the lapse-rates computed for each layer depend only upon the height, which is closely related to the logarithm of pressure, and the temperature. Equation (3) expresses the method of interpolation employed to compute the tropopause pressure when the lapse-rate was assumed to vary linearly with the logarithm of pressure.

$$\mathrm{PTROPE} = \exp\{\ln \mathrm{PCNTUP} + ((0.002 - \mathrm{XLAPUP})(\ln \mathrm{PCNTDN} - \ln \mathrm{PCNTUP}))/(\mathrm{XLAPDN} - \mathrm{XLAPUP})\} \quad (3)$$

TEST RESULTS:

For simplicity, the new model will be referred to as the $\gamma(p)$-model, while $\gamma(p)$+EXT and $\gamma(\ln p)$ will refer to: 1) the model variation with the 2000 m extension test mentioned in the introduction; and 2) the model variation wherein the lapse-rate varies linearly with the logarithm of pressure.

Each model was tested for both a ± 50 mb and a ± 25 mb difference between the computed tropopause and the lowest level radiosonde reported tropopause pressure (LLRRTP) for each radiosonde. As illustrated in Figure 2, there was virtually no difference between the $\gamma(p)$ and the $\gamma(\ln p)$ models for the ± 50 mb pressure window about the LLRRTP. Yet, when the more stringent tolerance of ±25 mb is used, the $\gamma(\ln p)$ model performs better, albeit marginally, averaging approximately 1% more computed tropopause pressures that are within 25 mb of the LLRRTP (See Figure 3). On the basis of that performance, only the $\gamma(\ln p)$ model was retained for comparison with the $\gamma(p)$+EXT and the operational models.

The test described above was applied in comparing the $\gamma(\ln p)$, the $\gamma(p)$+EXT ant the operational models, with the result being a more than 10% improvement of the $\gamma(\ln p)$ model over the operational model and an approximately 5% improvement over the $\gamma(p)$+EXT model for a ±50 mb pressure window (See Figure 4). When a pressure window of ±25 mb was used, the result was even more dramatic. The $\gamma(\ln p)$ model displayed an approximately 25% to more than 30% improvement over the operational model while maintaining an approximately 5% improvement over the $\gamma(p)$+EXT model (See Figure 5). The results of RMSE and bias error statistics are illustrated in Figures 6 and 7, respectively.

Finally, as displayed in Figure 8, a count was kept of the number of times that the absolute difference between the LLRRTP and model computed tropopause pressures was greater than or equal to 100 mb. No attempt was made to screen those soundings which contained apparent transmission or recording errors (e.g. a 40° change in temperature over 50 mb). However, because operational use of the models would be for forecast field tropopauses only, there could not be any transmission errors. Moreover, the greater number of differences generated that were equal to or in excess of 100 mb was the result of disagreement between the LLRRTP and the models' computed tropopause pressures rather than the result of errors in the sounding data. Figure 8 indicates that the tropopause pressures computed by the $\gamma(\ln p)$ model were within 100 mb of the LLRRTP much more often than either the $\gamma(p)$+EXT or the operational models.

CONCLUSION AND REMARKS:

The test results presented in the previous section provide a clear indication that the $\gamma(\ln p)$ model is exceedingly more accurate than the operational model with respect to the LLRRTP. As an example, the RMSE statistics from the $\gamma(\ln p)$ model showed an approximately 20 mb improvement over the operational model, while the operational model compared to the $\gamma(\ln p)$ model had more than three times the number of computed tropopauses that differed from the LLRRTP by 100 mb or more.

Of the three models tested, the $\gamma(p)$+EXT model conformed most closely to the WMO definition. However, unlike in the WMO definition wherein significant as well as mandatory level data is incorporated in determining the tropopause pressure, the $\gamma(p)$+EXT model used only mandatory level data. This difference was probably responsibly, at least in part, for the $\gamma(p)$+EXT model not performing better than it actually did.

Finally, although the $\gamma(\ln p)$ model made only very modest improvements over the $\gamma(p)$ model, the linear relationship between the lapse-rate and the logarithm of pressure was probably more physically realistic than was the linear relationship between the lapse-rate and pressure.

REFERENCES:

1. Hughes, W.L: 1981, "Numerical Forecasts Of Tropopause And Maxwind", International Civil Aviation Organization, Area Forecast Panel Second Meeting, Montreal, 21st September to 9th October 1981.

2. Radiosonde Code, Federal Meteorological Handbook No. 4, U.S. Department of Commerce, 1976.

3. Resolution 21 of WMO EC-IX, recorded in WMO 508.

CAPTIONS

Figure 1. The basic tropopause modelling assumptions mentioned in the section on model description.

Figure 2. On average, approximately 428 cases per day were used to determine the percentage of computed tropopause pressure values that fell within ± 50 mb of the lowest level radiosonde reported tropopause pressure (LLRRTP). The $\gamma(p)$ and $\gamma(\ln p)$ models display essentially no difference.

Figure 3. This figure is the same as Figure 2 except that a pressure window of ± 25 mb about the LLRRTP is used. The $\gamma(\ln p)$ model now displays a marginal improvement over the $\gamma(p)$ model.

Figure 4. On average, approximately 428 cases per day for the $\gamma(\ln p)$ and the $\gamma(p)$+EXT models and 411 cases per day for the operational model were used to determine the percentage of computed tropopause pressure values that fell within ± 50 mb of the LLRRTP. The graph shows that the $\gamma(\ln p)$ model improves over both the $\gamma(p)$+EXT and the operational models.

Figure 5. This figure is the same as Figure 4 except that a pressure window of ± 25 mb about the LLRRTP is used. Once again the performance of the $\gamma(\ln p)$ model is clearly superior to that of the other two models.

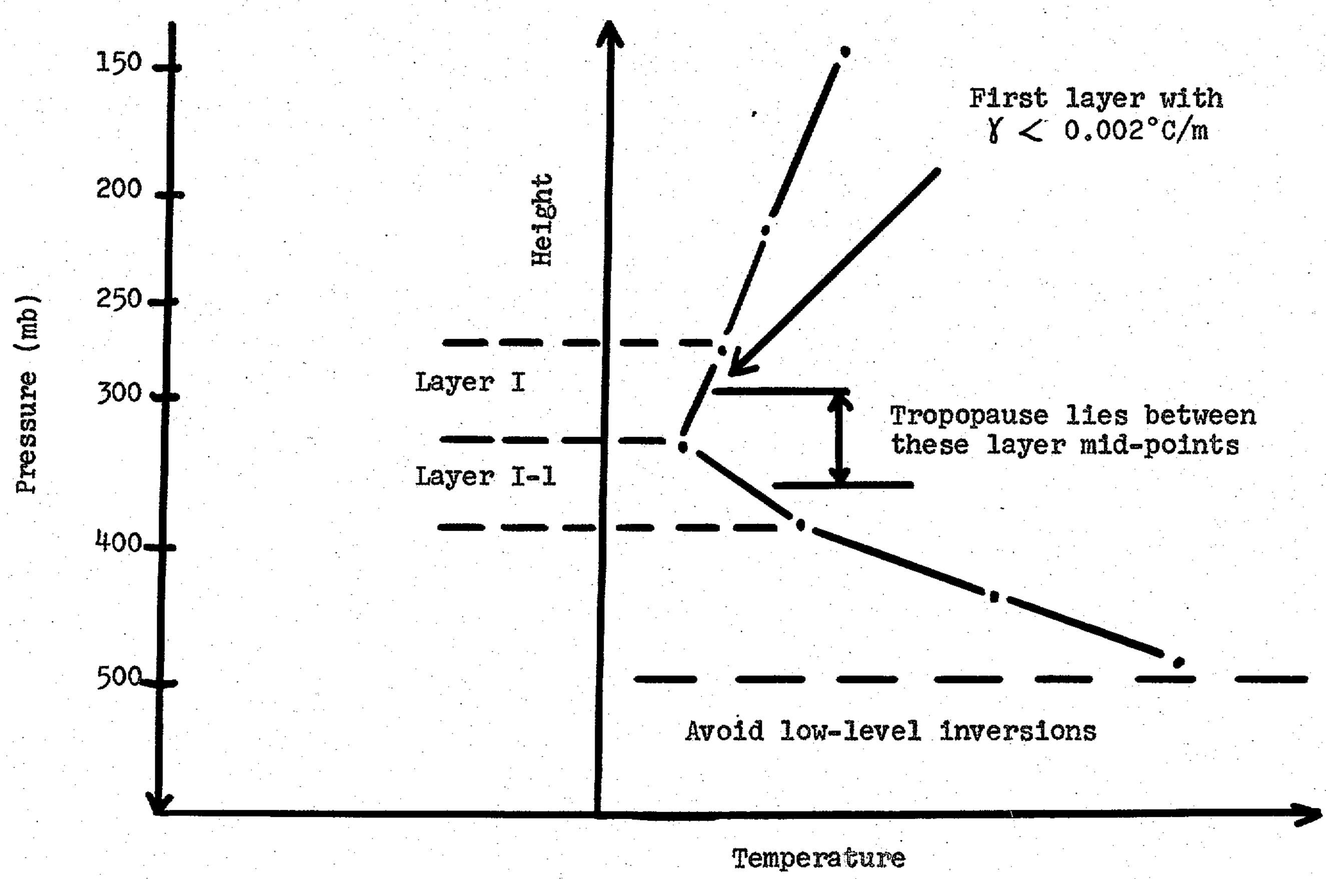
Figure 6. An average of approximately 428 cases went into determining the RMSE for the $\gamma(\ln p)$ and the $\gamma(p)$+EXT models, while an average of approximately 411 cases were used to compute the RMSE for the operational model. The $\gamma(\ln p)$ model clearly indicates a lower RMSE than either of the two other models.

Figure 7. The bias error was determined as follows:

Σ(LLRRTP − PTROPE)/N, where N = the number of cases.

The γ(ln p) model shows a small but decided negative bias. This indicates a tendency to compute the tropopause at a slightly higher pressure than the LLRRTP.

Figure 8. The total number of radiosonde reports for each of the time periods displayed wherein the absolute difference between the LLRRTP and the computed tropopause is greater than or equal to 100 mb. The γ(ln p) model averaged far fewer of these cases than either of the other two models.



150
200
250
300
400
500
Pressure (mb)
Height
First layer with
$\gamma < 0.002°C/m$
Layer I
Layer I-1
Tropopause lies between
these layer mid-points
Avoid low-level inversions
Temperature

Figure 1.

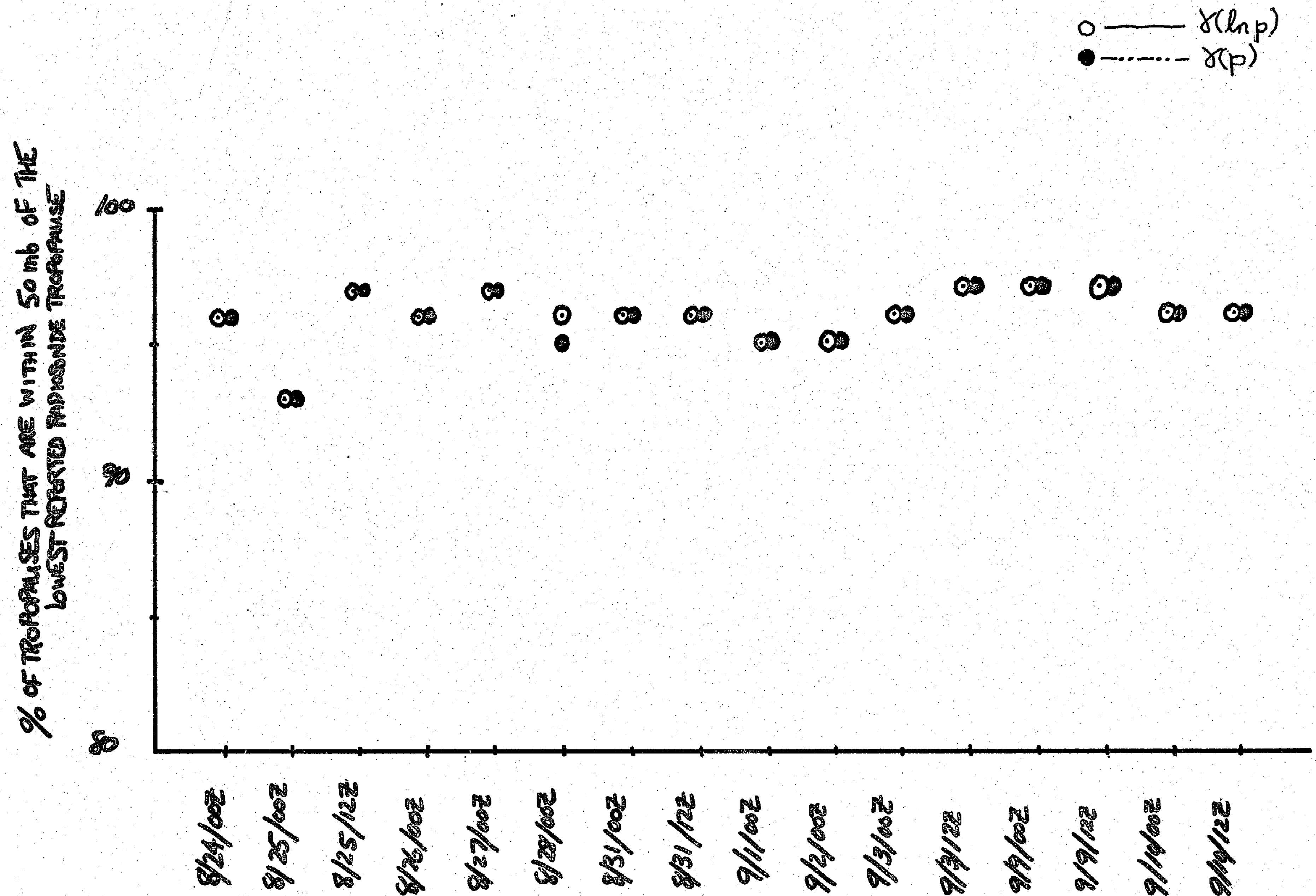
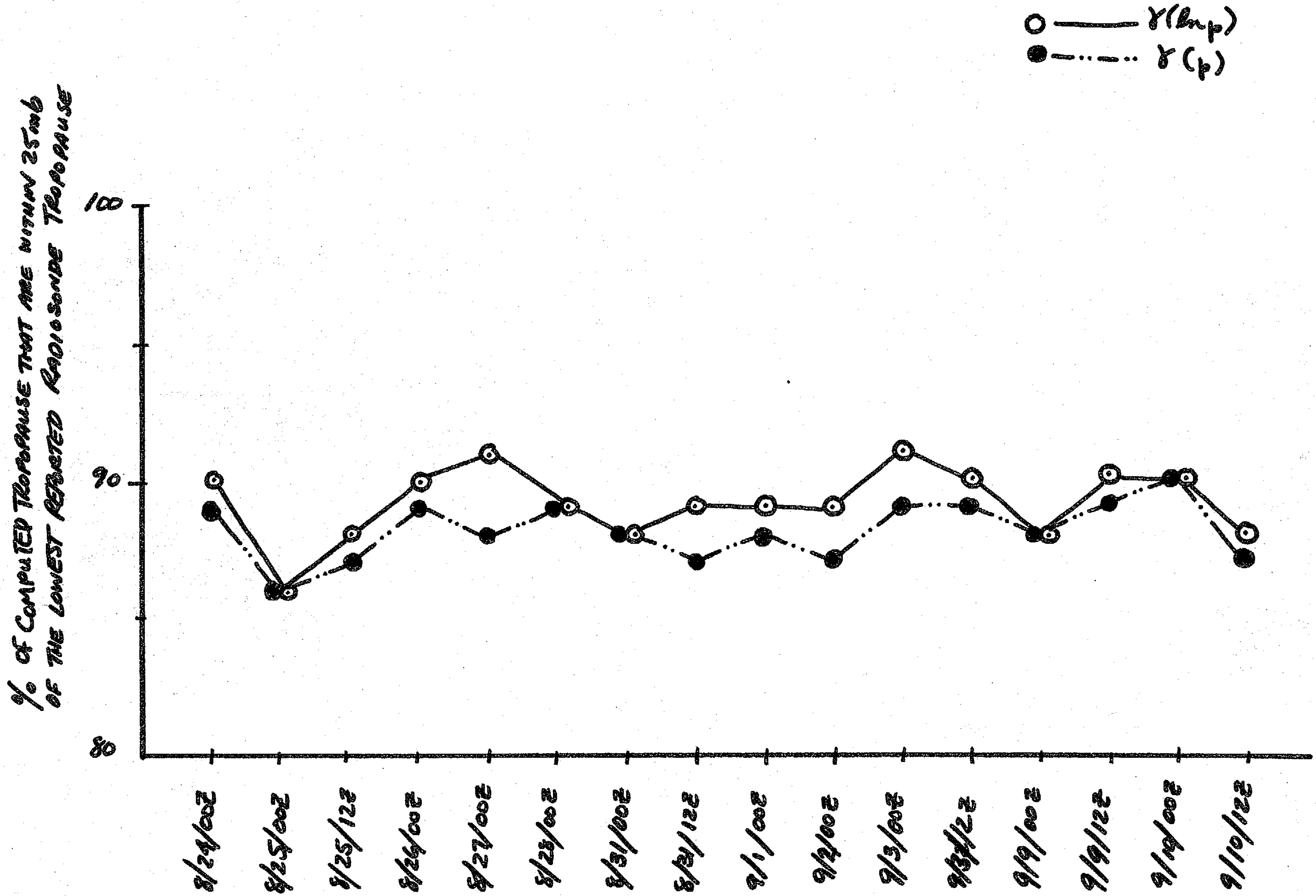
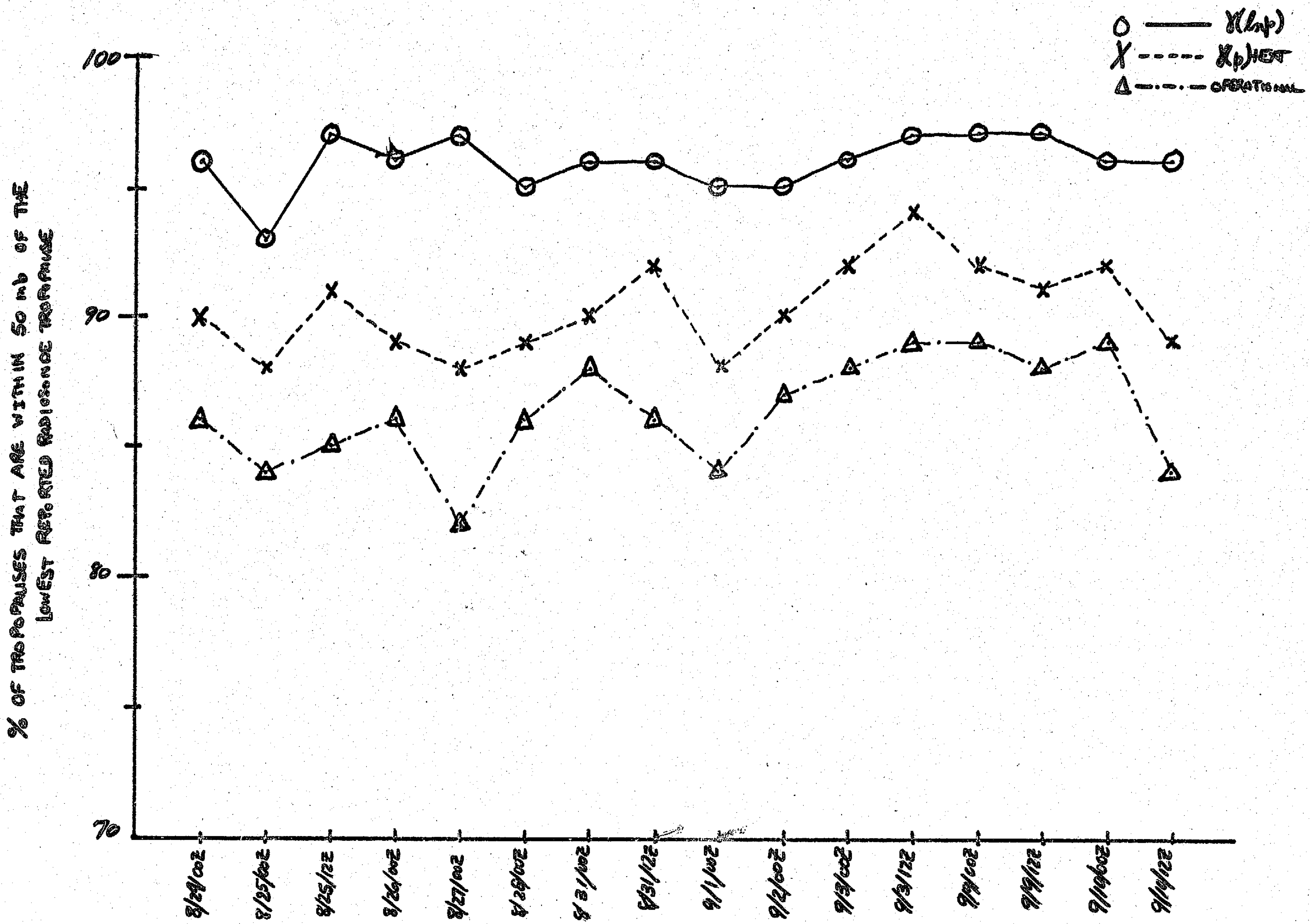

○ —— γ(ln p)
● -·-·-·- γ(p)
% OF TROPOPAUSES THAT ARE WITHIN 50 mb OF THE LOWEST REPORTED RADIOSONDE TROPOPAUSE
100
90
80
8/24/00Z
8/25/00Z
8/25/12Z
8/26/00Z
8/27/00Z
8/28/00Z
8/31/00Z
8/31/12Z
9/1/00Z
9/2/00Z
9/3/00Z
9/3/12Z
9/9/00Z
9/9/12Z
9/10/00Z
9/10/12Z

Figure 2.



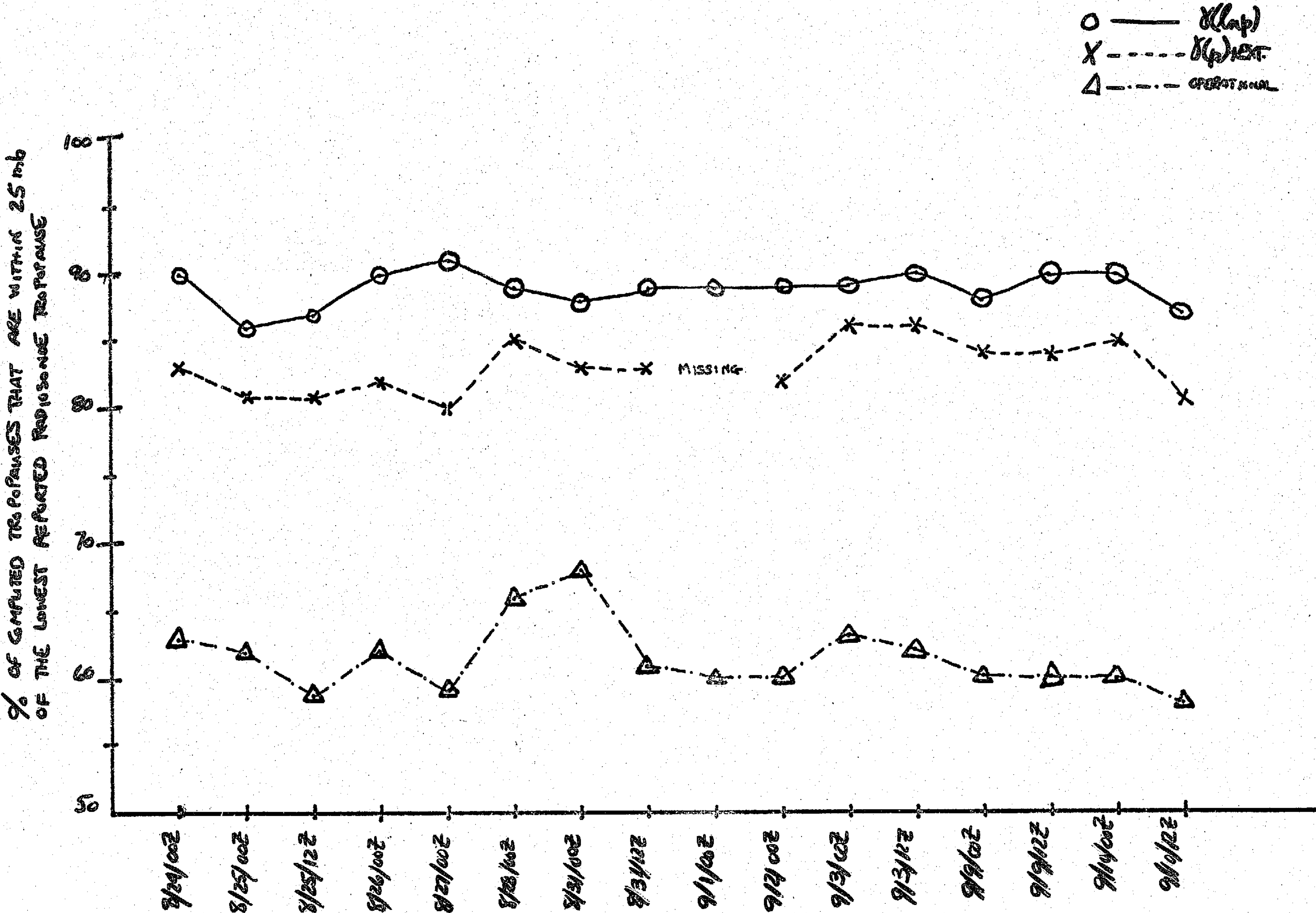
γ(ln p)
γ(p)
% OF COMPUTED TROPOPAUSE THAT ARE WITHIN 25 mb OF THE LOWEST REPORTED RADIOSONDE TROPOPAUSE
100
90
80
8/24/00Z
8/25/00Z
8/25/12Z
8/26/00Z
8/27/00Z
8/28/00Z
8/31/00Z
8/31/12Z
9/1/00Z
9/2/00Z
9/3/00Z
9/3/12Z
9/9/00Z
9/9/12Z
9/10/00Z
9/10/12Z

Figure 3.



% OF TROPOPAUSES THAT ARE WITHIN 50 mb OF THE LOWEST REPORTED RADIOSONDE TROPOPAUSE
100
90
80
70
8/24/00Z
8/25/00Z
8/25/12Z
8/26/00Z
8/27/00Z
8/28/00Z
8/31/00Z
8/31/12Z
9/1/00Z
9/2/00Z
9/3/00Z
9/3/12Z
9/4/00Z
9/4/12Z
9/10/00Z
9/11/12Z
OPERATIONAL

Figure 4.



OPERATIONAL
% OF COMPUTED TROPOPAUSES THAT ARE WITHIN 25 mb OF THE LOWEST REPORTED RADIOSONDE TROPOPAUSE
100
90
80
70
60
50
MISSING

Figure 5.

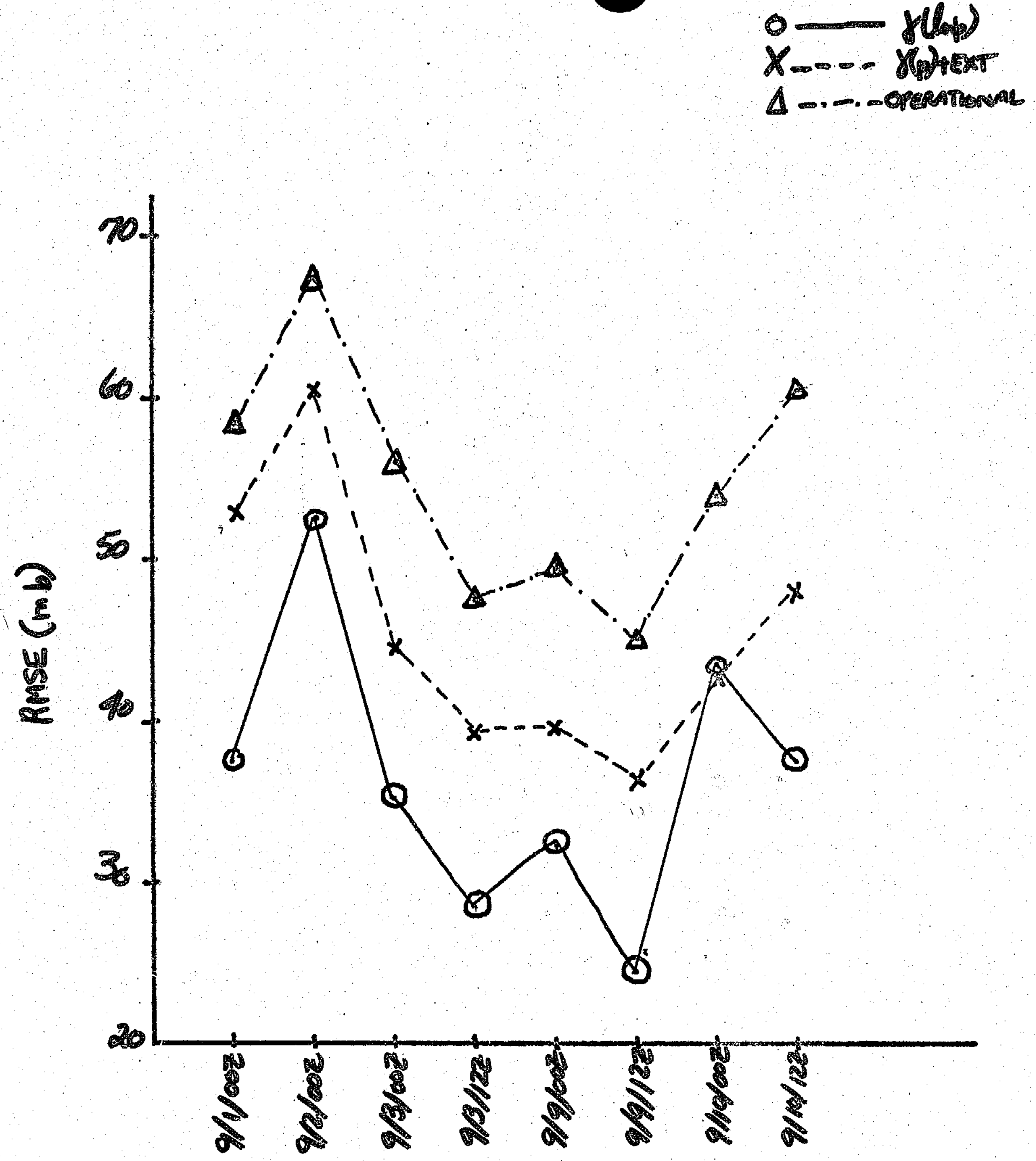
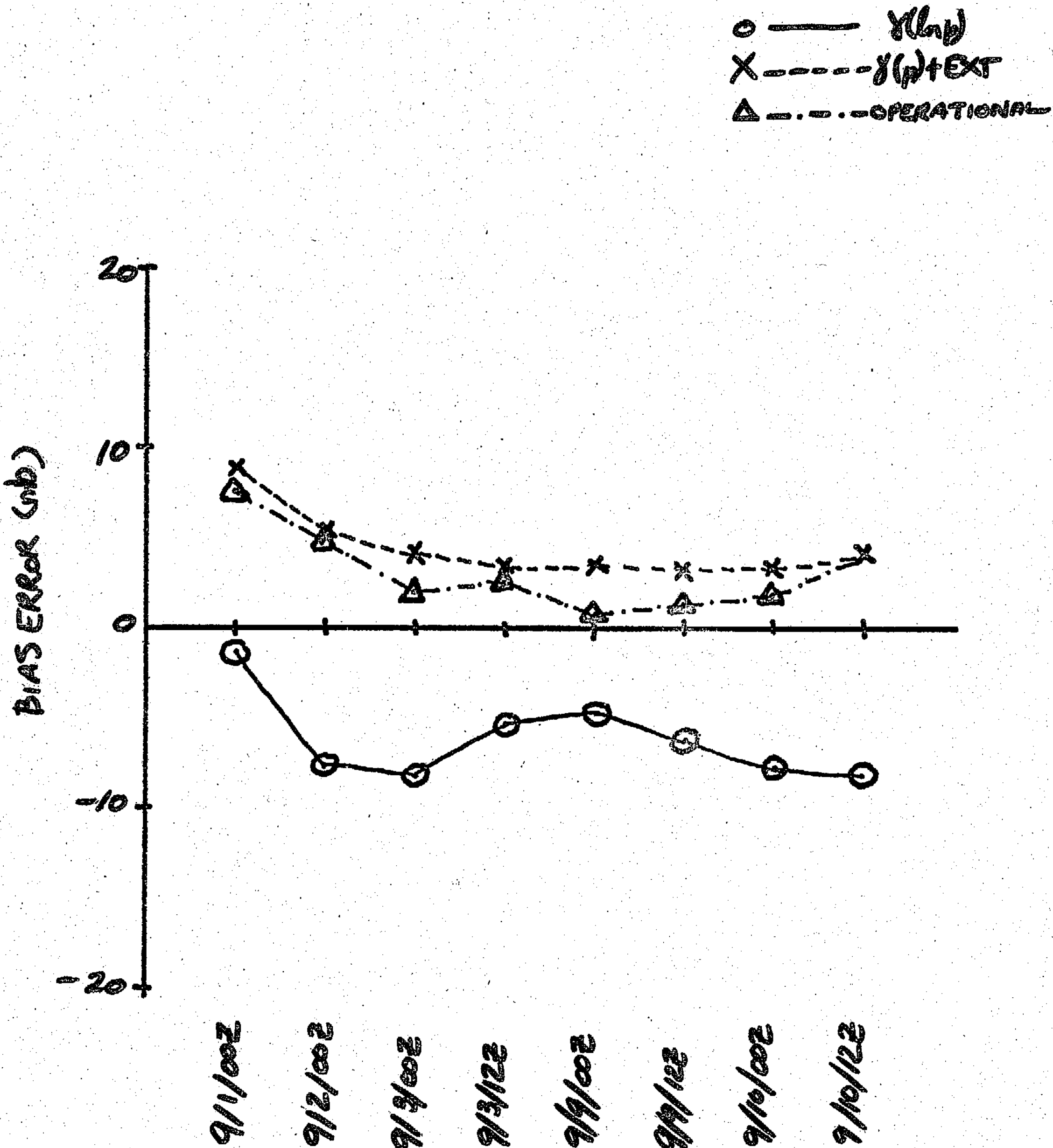

γ(lnp)
γ(p)+EXT
OPERATIONAL
RMSE (mb)
70
60
50
40
30
20
9/1/00Z
9/2/00Z
9/3/00Z
9/3/12Z
9/4/00Z
9/4/12Z
9/10/00Z
9/10/12Z

Figure 6.



γ(ln p)
γ(p)+EXT
OPERATIONAL
BIAS ERROR (mb)
20
10
0
-10
-20
9/1/00Z
9/2/00Z
9/3/00Z
9/3/12Z
9/4/00Z
9/9/12Z
9/10/00Z
9/10/12Z

Figure 7.

Total # of soundings with the
absolute value of (LLRRTP-PTROPE)
greater than or equal to 100 mb

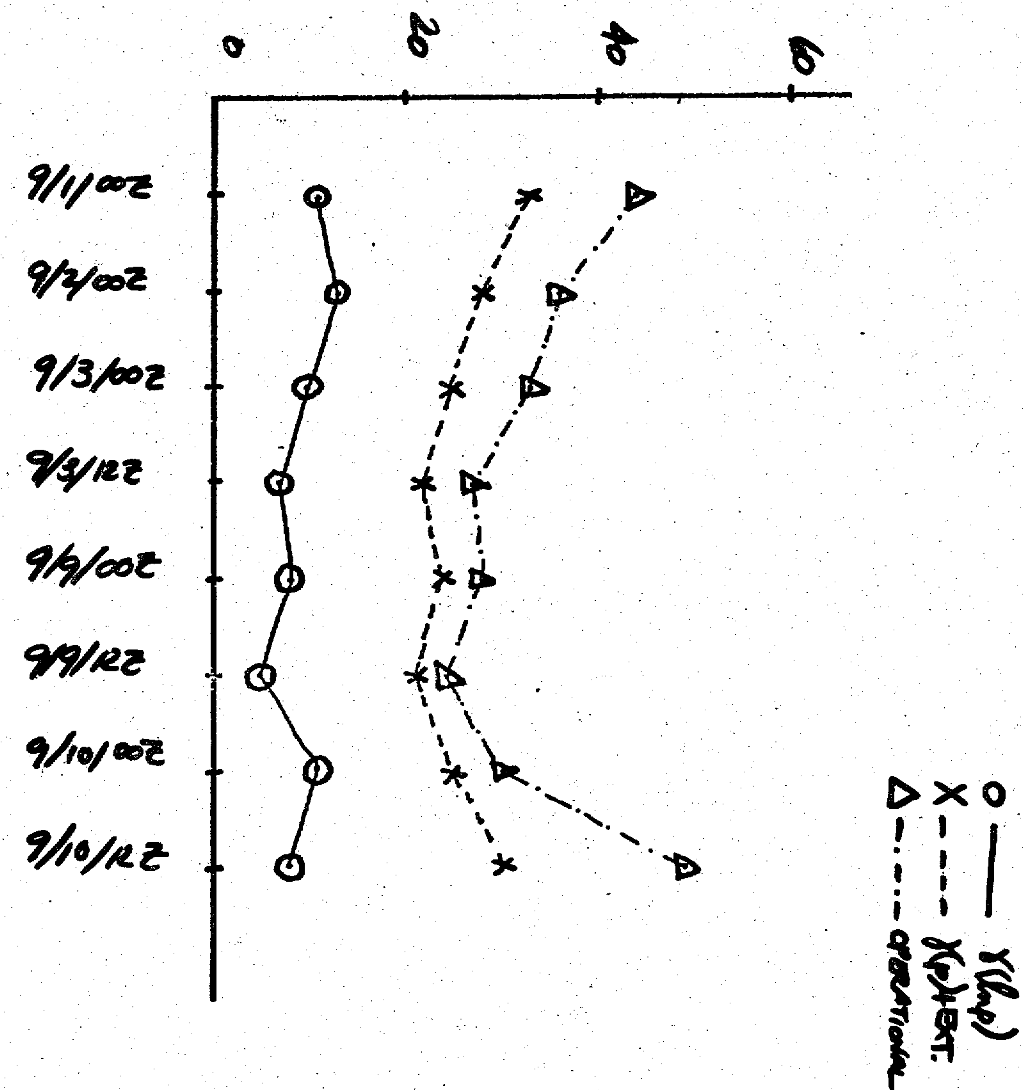


Figure 8.